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THEORETICAL BOUNDING RELATIONS FOR VOID FRACTION IN BUBBLING FLUIDIZED BEDS

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Abstract—Two-phase theory is used to derive limiting expressions for bubble fraction, voidage, and expansion ratio in aggregatively fluidized beds. These relations are compared with available data and shown to generally bound the range of bed expansion in both large-particle and small particle beds.

INTRODUCTION

The dynamics of gas voids or bubbles rising and growing within the solids/gas emulsion are today known to define the operating characteristics of most large-particle and many small-particle fluidized beds. Although the behavior of single, isolated bubbles has been explored in numerous analytical and experimental studies, the more complex characteristics of multiple-bubble-beds have not been thoroughly addressed. The diameter, rise velocity, concentration of bubbles and even total void fraction have generally been determined from empirical relations for similar bed parameters, leaving major discrepancies unresolved.

The expansion of the solids/gas emulsion in a fluidized medium, as well as its variation with gas flow rate and other relevant parameters, establishes the height of the bed container and is of critical importance to its design. When the total mass of the solids is fixed, expansion of the fluidized medium results from increased gas fraction and in bubbly or aggregative fluidization this expansion can be related to the emulsion voidage and bubble fraction prevailing in the bed. In succeeding sections, two-phase theory, in combination with appropriate assumptions about bubble rise velocity, will be used to derive bounding relations for the void fraction in both small and large particle beds. These relations will then be used to bound the anticipated void fraction for several distinct parametric ranges and the results compared with experimental data.

THEORETICAL ANALYSIS

Two-phase theory

While many diverse methods of summing the distinct voidage components in a bubbling fluidized bed can be proposed, the basic two-phase flow relations offer a convenient structure for any such determination. In two-phase theory, the emulsion, consisting of both gas and solid particles, is assumed to be a single homogenous phase at the minimally fluidized condition and the bubble gas is viewed as the dispersed second phase. This theory is commonly employed in the analysis of small-particle aggregatively fluidized beds (Davidson & Harrison 1963; Kunii & Levenspiel 1968) and when properly structered has been shown by Bar-Cohen *et al.* (1977) to apply as well to large particle fluidization. However, in very fine particle systems, involving solids belonging to Geldart's type A (Geldart 1978), the emulsion phase is frequently expanded well beyond the value at minimum fluidization and this category will, thus, be excluded from this discussion.

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Following Kunii & Levenspiel (1968) and neglecting wall effects, the average bed voidage, ϵ, can be expressed as a sumation of emulsion voidage and bubble voidage, the former assumed to equal ϵ_{mf} and the latter to equal unity. Thus

$$\epsilon = \epsilon_{mf}(1-\delta)+\delta \qquad [1]$$

where ϵ_{mf} is the bed voidage at minimum fluidization and δ is the bubble fraction. The bubble fraction can itself be determined from gas flow continuity considerations which dictate that total bed gas flow equal the sum of emulsion flow and bubble flow.

In examining gas flow continuity it is convenient to distinguish between the flow regime in which bubbles rise more slowly than the interstitial gas velocity in the emulsion and the flow regime in which bubbles rise more quickly than the interstitial gas. The former category may be defined as the "slow-bubble" regime and is generally encountered in large particle fluidization where the minimum fluidization velocity is high. Alternately, the "fast bubbles" are typical of small particle systems where, as a consequence of the relatively low minimum fluidization velocity, the rise rate of even a small diameter bubble can exceed the interstitial gas velocity.

In the slow-bubble regime, the continuity relation takes the form (Kunii & Levenspiel 1968):

$$U = U_{mf}(1-\delta)+(u_b+3U_{mf})\delta \qquad [2]$$

where U is the superficial velocity of the fluidizing gas, U_{mf} the minimum fluidization value and u_b the bubble rise velocity. Some uncertainty exists as to the range of applicability of this relation and especially the effects of neighboring bubbles in multiple-bubble flow. A recent analysis by Bar-Cohen *et al.* (1977) of large particle data obtained by Cranfield & Geldart (1974) suggests, however, that [2] is valid for multiple-bubbles throughout at least most of the slow bubble regime, i.e. for $0 < u_{br} < u_f$ (where u_{br} is the rise velocity of an isolate bubble and u_f the interstitial gas velocity taken to equal U_{mf}/ϵ_{mf}) and is thus applicable to the present analysis.

For very fast bubbles, i.e. when u_{br} is more than five times the interstitial velocity and the clouds surrounding the bubbles are negligible in size, the superficial gas velocity can be approximated by (Kunii & Levenspiel 1968):

$$U = U_{mf}(1-\delta)+u_b\delta. \qquad [3]$$

The gas continuity relation for either slow or fast bubbles necessarily involves the bubble rise velocity, u_b. Extensive studies of isolated bubbles have established that the rise velocity of a single bubble in an otherwise undisturbed gas fluidized medium is given by (Davidson & Harrison 1963):

$$u_{br} = 0.711\,(gd_V)^{1/2} \qquad [4]$$

where d_V is the diameter of the equivalent spherical bubble, and that the rise velocity of a bubble in a freely bubbling bed can be approximated by:

$$u_b = u_{br} + U - U_{mf}. \qquad [5]$$

Detailed measurements of u_b for individual bubbles in bubble swarms by Werther (1975, 1977) and Whitehead *et al.* (1967) suggest that bubble velocities are often higher than given by [5]. But, in the absence of a more precise relation, this simple formulation will be used throughout the present discussion.

When u_b from [5] is inserted into [2] and [3] and these are individually combined with [1], the desired voidage relations are found as:

$$\epsilon_{sb} = \epsilon_{mf} + (1 - \epsilon_{mf})(U - U_{mf})/(U + u_{br} + U_{mf}) \qquad [6]$$

for $u_{br} < u_f$, and

$$\epsilon_{fb} = \epsilon_{mf} + (1 - \epsilon_{mf})(U - U_{mf})/(U + u_{br} - 2U_{mf}) \qquad [7]$$

for $u_{br} < 5u_f$, where ϵ_{sb} is the bed voidage in the slow bubble regime and ϵ_{fb} the voidage in the fast bubble regime.

Equations [6] and [7] reveal bed voidage in both the slow and fast bubble regimes to be dependent on bubble rise velocity which, due to bubble growth (Werther 1977; Cranfield & Geldart 1974), generally increases with height. Many empirical correlations for bubble diameter exist in the literature (e.g. Darton *et al.* 1977) but there is as yet no acceptable theory of bubble growth nor a generalized bubble size correlation and, consequently, it is difficult to precisely determine the void fraction in a bubbling bed.

The relative bubble rise velocity is, however, constrained to vary within prescribed bounds—between zero and the interstitial velocity for slow bubbles and between the interstitial velocity and the slug velocity for fast bubbles. It is, thus, possible to identify several characteristic bounds for fluidized bed voidage, each associated with a limiting value of bubble rise velocity.

Slow bubble regime

The maximum bubble rise velocity in the slow bubble regime is, by definition, equal to the interstitial gas velocity, u_f, itself taken equal to U_{mf}/ϵ_{mf} by the common two-phase assumption. A lower-bound on bed voidage in this regime can thus be obtained by setting u_{br} equal to U_{mf}/ϵ_{mf} in [6] to yield (with $U' \equiv U/U_{mf}$)

$$\epsilon_{tr} = \epsilon_{mf} + (1 - \epsilon_{mf})(U' - 1) \div (U' + 1 + 1/\epsilon_{mf}), \qquad [8]$$

where ϵ_{tr} is the bed voidage associated with u_{br} equal to u_f. Alternately, the voidage upper-bound can be derived from [6] by allowing the relative bubble rise velocity to approach zero, as might be appropriate in the region adjacent to the gas distributor. Thus

$$\epsilon_{\text{stat}} = \epsilon_{mf} + (1 - \epsilon_{mf})(U' - 1)/(U' + 1), \qquad [9]$$

where ϵ_{stat} is the bed voidage associated with stationary bubbles. Using similar reasoning the bounding expressions for bubble fraction and expansion ratio, shown in table 1, can be derived.

Table 1. Bounding bubble fraction and expansion ratio expression for fluidized beds Bubble fraction, δ

$\delta_{\text{stat}} = (U' - 1)/(U' + 1)$	
$\delta_{\text{trans}} = (U' - 1)/(U' + 1 + 1/\epsilon_{mf})$	
$\delta_{\text{slug}} = 1 - [1 + (U - U_{mf})/0.35\sqrt{gD}]^{-1}$	(Hovmand & Davidson 1971)

where δ_{stat} in the bubble fraction associated with stationary bubbles; δ_{trans} bubble fraction associated with bubbles in transition from the slow to fast bubble regime; δ_{slug} is the bubble fraction in the slugging regime; and D is the bed diameter.

Bed height, H:

$$H_{stat} = H_{mf}(U' + 1)/2$$

$$H_{trans} = H_{mf}(U' + 1 + 1/\epsilon_{mf})/(2 + 1/\epsilon_{mf})$$

$$H_{slug} = H_{mf}[1 + (U - U_{mf})/0.35\sqrt{gD}], \qquad \text{(Hovmand \& Davidson 1971)}$$

where H_{stat} is the bed height associated with stationary bubbles, H_{mf} the bed height at minimum fluidization, H_{trans} bed height associated with transition from the slow to fast bubble regime and H_{slug} the bed height in the slugging regime.

Fast bubble regime

The onset of slugging in fluidized beds represents the end of simple bubbling behavior and in the absence of other flow regime transitions sets an upper-limit on the relative rise velocity of the bubble. Based on the maximum bed expansion ratio in the slugging regime found by Hovmand & Davidson (1971) it is possible to establish the desired lower bound for bed voidage in the fast bubble regime, as

$$\epsilon_{slug} = 1 - (1 - \epsilon_{mf})/[1 + (U - U_{mf})/0.35(gD)^{1/2}]. \qquad [10]$$

The minimum bubble velocity in the fast bubble regime is established by the transition criteria, i.e. $u_{br} = U_{mf}/\epsilon_{mf}$, and consequently, ϵ_{tr} sets the upper bound on fast bubble voidage.

COMPARISON WITH DATA

Slow bubble regime

In the large-particle fluidized bed study by Cranfield & Geldart (1974), local bubble frequency, $f_{C\&G}$, (the frequency with which bubbles strike a point probe) and bubble diameter were measured by independent means over a significant parametric range. These experimental values can be used to determine the local bubble fraction in the bed by equating the product of the volume of each bubble and the frequency with which bubbles traverse a given level, with the gas flow rate through the area-fraction of the bed occupied by bubbles, i.e.

$$f_\ell(\pi d_V^3/6) = \delta A u_b, \qquad [11]$$

where f_ℓ is the level frequency and A is the cross-sectional area of the bed. The Cranfield & Geldart (1974) frequency, $f_{C\&G}$, is related to level frequency by the ratio of bed cross-sectional area to the projected area of a single bubble, i.e.

$$f_\ell = f_{C\&G}A/\pi d_V^2/4. \qquad [12]$$

Combining [11] and [12], δ for the Cranfield & Geldart study is found to equal

$$\delta = z f_{C\&G} d_V/3u_b. \qquad [13]$$

Typical δ values calculated from [13] for two different tests in the slow bubble regime, with u_{br}/u_f ranging from 0.4 to 0.67 and 0.58 to 0.94, respectively, are shown in figure 1. As transition is approached, the bubble fraction closely approaches the transition value of δ. The data are seen to display the anticipated variation with bed height and fall within the bounds established by the transition and stationary-bubble δ expressions.

Similarly, the average bubble fraction recorded in the nineteen Cranfield & Geldart (1974)

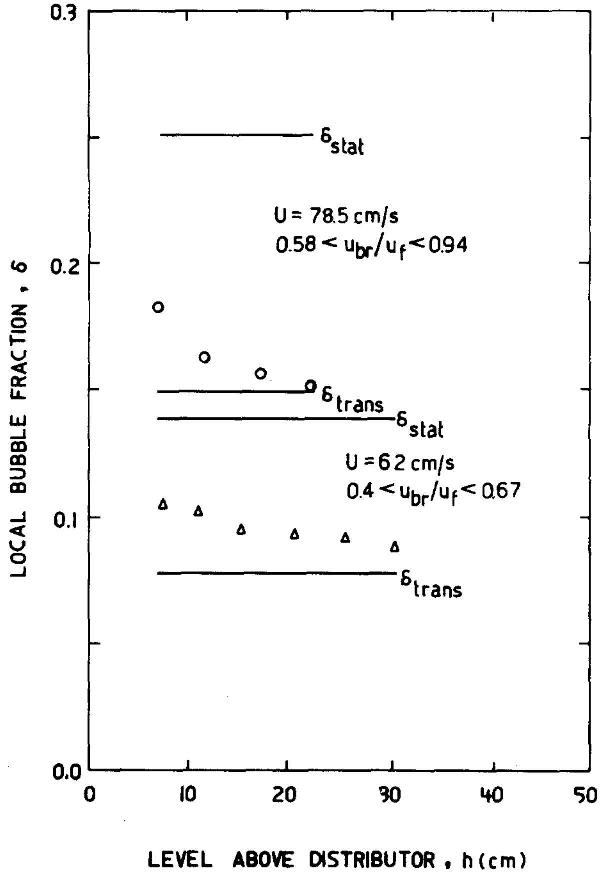

Figure 1. Variation of local bubble fraction with level above distributor. Data of Cranfield & Geldart (1974). [$d_p = 1760\ \mu$m, $U_{mf} = 47$ cm/s, $\epsilon_{mf} \simeq 0.55$, Bed: 61 cm × 61 cm].

data runs are found to follow the trends suggested by [6] and to be properly bounded by stationary and transition bubble fraction relations.

The paucity of consistent slow-bubble results in the literature makes it most difficult to validate theoretical relations for slow-bubble fluidized beds and often dictates analytical comparisons with incomplete data. In particular, for the bubbling study by McGrath & Streatfield (1971), where u_{br}/u_f varied from 0.24 to 0.68 the minimum fluidization voidage, ϵ_{mf}, is not reported, and many of the bubbles approach or exceed the small dimension of the rectangular cross-section bed employed in the experiments. Nevertheless, when, as shown in figure 2, expansion ratio values are compared with the previously derived analytical expressions with an assumed value of $\epsilon_{mf} = 0.41$, nearly all the data are found to lie between the stationary and transition bounds.

The large particle data obtained by Canada *et al.* (1976), for relatively high gas flow rates in 30.5 and 61 cm wide beds filled with 2600 μ glass particles, offer a further opportunity for examining the applicability of the bounding relations. Much of the operation of these laboratory fluidized beds was in the slugging and the post-slugging turbulent regimes and bubble sizes were not reported. However, due to the high minimum fluidization velocity encountered in these experiments, approx. 1.3 m/s, slow bubble dynamics might be anticipated to dominate bed behavior.

Canada *et al.* (1976) correlated their results by an expression of the form

$$U/\epsilon = U + (1 - \epsilon_{mf})/\epsilon_{mf} U_{mf}. \qquad [14]$$

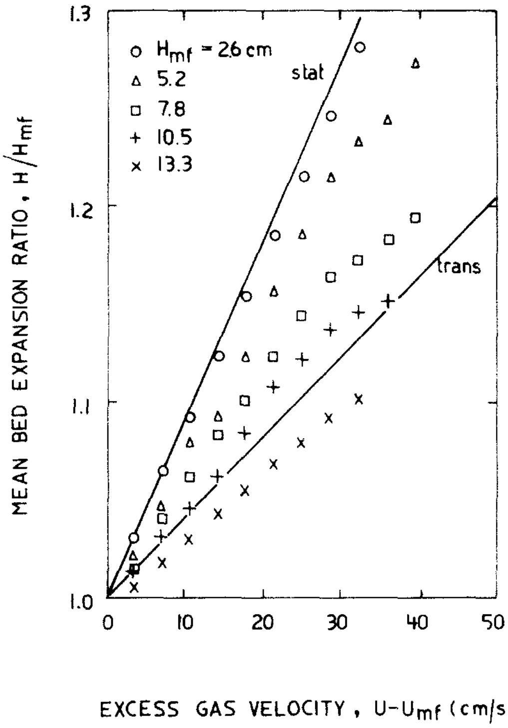

Figure 2. Variation of height ratio with velocity ratio. Data of McGrath & Streatfield (1971). [$d_p = 1540\ \mu$m, $U_{mf} = 55$ cm/s, $\epsilon_{mf} = 0.41$ (estimated), Bed: 15.2 cm × 30.5 cm].

Equation [14] and the Canada *et al.* large particle data are shown in figure 3 along with the stationary and transition bounds previously derived. Equation [14] is seen to be nearly identical with the stationary-bubble bound for the 31 cm bed and indistinguishable from this upper-bound on slow-bubble voidage in the 61 cm bed. Furthermore, although Canada *et al.* observed aggregative fluidization only at gas velocities below 2 m/s, or $U/U_{mf} < 1.5$, nearly all the data points in the range $1 < U/U_{mf} < 5$ appear to fall within the slow-bubble voidage bounds, with a pronounced clustering near the presumed upper-bound for bed voidage in the slow-bubble regime. It is somewhat surprising that this expression appears to apply to voidage values obtained in the post-slugging turbulent regime, but may point to wider applicability of the derived slow-bubble voidage relations than initially supposed.

It is important to note that the very "slow" nature of much of the Canada *et al.* data may explain the close proximity of their correlation to the stationary-bubble bound. This relationship may not, however, apply to significantly wider beds, where values of u_{br}/u_f could approach and exceed unity, and caution should, therefore, be exercised in extrapolating the results reported by Canada *et al.*

Fast bubble regime

In evaluating fast bubble data, it is important to note that, during transition from bubbly to slug flow, the bubble rise velocity can exceed the values associated with the slug by approx. 30 per cent and, furthermore, that an isolated bubble of approx. one-quarter the column diameter can be expected to display a rise velocity equal to the slug velocity (Werther 1975, Hovmand & Davidson 1971).

A typical fast-bubble comparison is shown in figure 4 where data, obtained by Geldart (1967) for 101 μm sand particles, are seen to generally lie between the loci of the transition expression and the slug relation. Despite the absence of slugs in the bed, the reported voidage ratio approaches (and for two data points crosses) the slug relation. This behavior can be related to

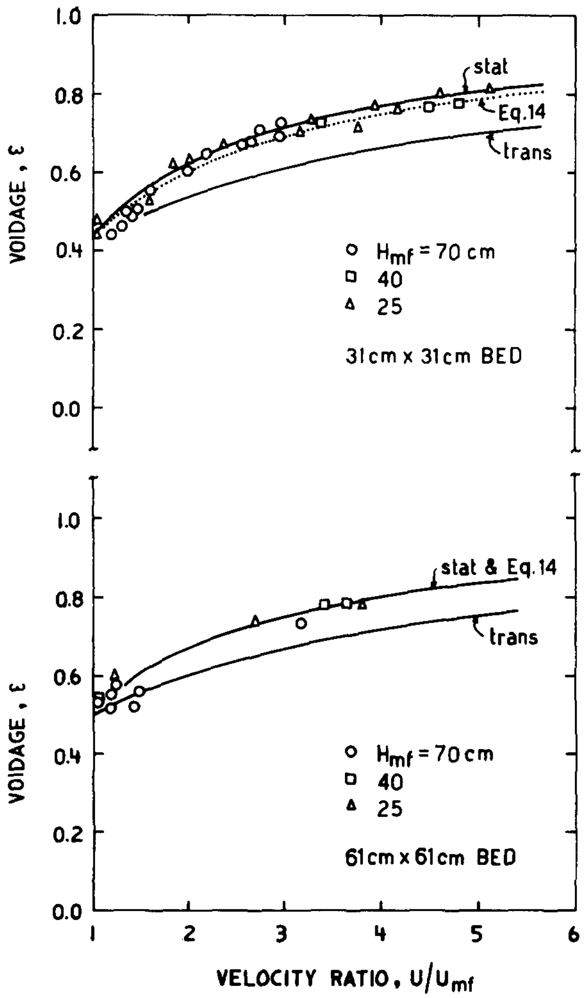

Figure 3(a). Variation of voidage with velocity ratio. Data of Canada *et al.* (1976). [d_p = 2600 μm, U_{mf} = 1.3 m/s, $\epsilon_{mf} \simeq 0.43$, Bed: 30.5 cm × 30.5 cm].

Figure 3(b). Variation of voidage of velocity ratio. Data of Canada *et al.* (1976). [d_p = 2600 μm, U_{mf} = 1.3 m/s, $\epsilon_{mf} \simeq 0.50$, Bed: 61 cm × 61 cm].

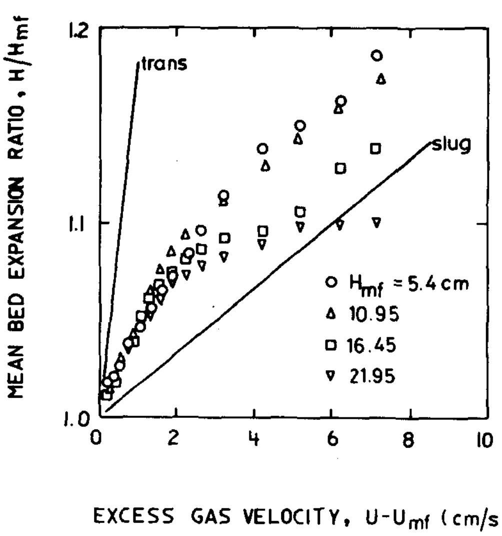

Figure 4. Variation of height ratio with excess gas velocity. Data of Geldart (1967/68). [d_p = 101 μm, U_{mf} = 1.37 cm/s, ϵ_{mf} = 0.49, Bed: 30.8 cm dia.].

the possibility of a bubble velocity somewhat larger than the slug velocity in the bubbly to slug flow transition, as mentioned above.

Similar results are obtained when data from a later study by Geldart (1972) as well as an experimental investigation by de Groot (1967) are compared with the appropriate bounding expressions. Alternately, examination of experimental results reported by Fryer & Potter (1976) and Gibbs & Perry (1968) for small particle fluidization show the bed expansion values to scatter above and below the slug relation. Since the expansion ratio for a slugging bed, shown in table 1, is considered to represent the maximum bed expansion in slug flow, it might be anticipated that the average bed height reported will lie below this curve by not more than a single slug diameter or approximately one-half the column diameter. This condition is met by most of the "low" data points mentioned above.

APPLICATION TO FLUIDIZED BED DESIGN

The desire to provide the designer of fluidized beds with analytical expressions for bed expansion motivated much of the present development. The bounding relations obtained and validated in previous sections can be applied directly to this task, once it is known in which regime the fluidized bed is likely to operate.

Slow bubble dynamics can be expected to dominate the behavior of shallow, large particle fluidized beds. If an estimate of maximum bubble size based on one of the available bubble size correlations, notably that due to Cranfield & Geldart (1974), suggests that the maximum bubble rise velocity is less than the interstitial gas velocity, U_{mf}/ϵ_{mf}, the bed can be presumed to operate in the slow bubble regime. For very small values of u_{br}, the stationary-bubble limit can be expected to provide a close, first-order prediction of bed expansion. When the average value of u_{br} is approximately equal to U_{mf}/ϵ_{mf}, the transition-bubble limit can be used and when the possible range of u_{br} values spans the entire slow-bubble regime, best results would be obtained by use of [6] with u_{br} set equal to $U_{mf}/2\epsilon_{mf}$.

In small particle beds, an initial estimate of the prevailing bubble diameter can be obtained by using empirical correlations, notably that due Werther (1976) or Darton *et al.* (1977). If the resulting estimate of u_{br} falls in the fast bubble regime, but close to the transition value, U_{mf}/ϵ_{mf}, the transition-bubble expression, can be used to estimate the bed expansion ratio. When, however, the bubble rise velocity approaches the velocity associated with a slug or when the bed is determined to be operating in the slugging regime, the bed voidage and/or expansion ratio should be determined by use of the slug relations.

Under circumstances that do not allow bubble diameter or rise velocity in the bed to be estimated, prediction of bed voidage and expansion ratio is made far more difficult. In a tall, narrow bed without internals or immersed heat-extraction pipes, bed diameter often constitutes a reasonable first estimate of maximum bubble size. In a shallow, wide bed without internals, maximum bubble size can perhaps be estimated by setting d_V equal to the bed depth, though an average of the dimensions or even use of the bed diameter may be appropriate if bubbling is very vigorous. Alternately, when arrays of heat extraction tubes are immersed in the fluidized medium, a first-order approximation of bed voidage can be obtained by assuming that the largest bubble diameter is equal to the spacing between the tubes. Once an estimate of bubble size is available, the relevant rise velocity can be determined from [2]. With u_{br} known, the most appropriate expression for bed expansion can be selected by following the procedure outlined above.

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